Addendum: Complete Harmonic Scale Hierarchy from Golden Ratio Pattern

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Related to: Original pre-registration of $\lambda_{13} = 0.856$ Mpc prediction
Status: Pre-registration addendum (published before Euclid DR1 observational data)

Abstract

This addendum extends the original pre-registered prediction of $\lambda_{13} = 0.856$ Mpc to include the complete harmonic scale hierarchy derived from the 6D geometric framework. The golden ratio pattern $\lambda_n = \lambda_2 \times \varphi^{n-2}$, where $\varphi = (1+\sqrt{5})/2 \approx 1.618$, predicts three independent scales in the cosmic web regime: $\lambda_{12} = 0.538$ Mpc, $\lambda_{13} = 0.856$ Mpc, and $\lambda_{14} = 1.385$ Mpc. This addendum is published before examining Euclid DR1 data to maintain pre-registration integrity.

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1. Introduction

The 3D+3D discrete spacetime framework predicts a hierarchy of characteristic scales arising from dimensional reduction of the 6D geometry. The original pre-registration specified $\lambda_{13} = 0.856$ Mpc as a target for cosmic web clustering analysis. This addendum documents the theoretical foundation for the complete harmonic progression, deriving predictions for adjacent scales λ_{12} and λ_{14}.

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2. Theoretical Framework

2.1 Harmonic Scale Progression

The 6D geometric framework with signature $(-,+,+,+,-,-)$ and two compactified temporal dimensions predicts a discrete hierarchy of characteristic scales. Given a fundamental scale λ_2 determined from galaxy rotation curve analysis ($\lambda_2 = 4.30$ kpc), the progression follows:

$$\lambda_n = \lambda_2 \times \varphi^{n-2}$$

where $\varphi = (1+\sqrt{5})/2$ is the golden ratio and n indexes the harmonic mode.

2.2 Derivation from 6D Geometry

The golden ratio emergence derives from the dimensional reduction procedure. The compactification of two temporal dimensions with radius R yields characteristic scales separated by factors of φ due to the self-similar structure of the geometric reduction. This is documented in detail in Paper I (Section 3) and Paper II (Section 4.2).

2.3 Connection to Original Prediction

The original pre-registered value $\lambda_{13} = 0.856$ Mpc corresponds to n = 13 in the harmonic series. This scale was selected based on expected sensitivity in cosmic web correlation functions at intermediate redshifts (z ~ 0.5-1.5). The present addendum extends the prediction to include adjacent modes.

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3. Extended Predictions

3.1 Three Harmonic Scales

The complete prediction for cosmic web scales includes:

Mode	Formula	Predicted Value	Physical Scale
n=12	$\lambda_2 \times \varphi^{10}$	0.538 Mpc	Sub-cluster
n=13	$\lambda_2 \times \varphi^{11}$	0.856 Mpc	Cluster
n=14	$\lambda_2 \times \varphi^{12}$	1.385 Mpc	Supercluster

3.2 Ratios Between Scales

The ratios between adjacent scales provide an additional test:

$\lambda_{13} / \lambda_{12} = \varphi = 1.618...$
$\lambda_{14} / \lambda_{13} = \varphi = 1.618...$

Observational detection of multiple scales with ratios consistent with φ within measurement uncertainties would provide stronger support than detection of a single scale.

3.3 Relationship to Previous Work

The fundamental scale $\lambda_2 = 4.30$ kpc was determined from SPARC galaxy rotation curve analysis (175 galaxies, RMS = 33 km/s). This represents the only empirical fit in the framework. All higher modes (n > 2) are theoretical predictions derived from λ_2 and the geometric reduction formula.

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4. Observational Implications

4.1 Euclid Survey Sensitivity

The Euclid Wide Survey is expected to probe correlation functions in the range 0.1-10 Mpc with high statistical precision. The three predicted scales span the range 0.5-1.4 Mpc, which falls within the optimal sensitivity regime for two-point correlation function measurements at z ~ 0.5-1.5.

4.2 Expected Signal Characteristics

Each harmonic scale λ_n is expected to produce an enhancement in the two-point correlation function $\xi(r)$ at the corresponding physical separation. The signal amplitude and width depend on the coupling strength between 3D and 6D sectors, which varies with scale according to the geometric reduction formalism.

4.3 Distinguishing from ΛCDM

The ΛCDM model predicts a smooth power-law decline in $\xi(r)$ at these scales, without distinct characteristic features. Detection of discrete peaks at the predicted locations with ratios consistent with φ would distinguish the 6D geometric framework from standard cosmology.

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5. Success and Falsification Criteria

5.1 Tiered Success Criteria

Tier 1 (Positive Detection): Identification of any single peak in $\xi(r)$ within ±10% of predicted values (λ_{12}, λ_{13}, or λ_{14}).

Tier 2 (Multiple Scale Detection): Identification of two or more peaks within ±10% of predicted values.

Tier 3 (Pattern Confirmation): Multiple peak detection with measured ratios consistent with $\varphi = 1.618 \pm 0.05$.

5.2 Falsification Criteria

The framework would be falsified by:

1. No detectable peaks at any predicted scale in Euclid DR1 with sufficient statistical power
2. Detection of peaks at substantially different locations (>20% deviation)
3. Detection of multiple peaks with ratios inconsistent with φ (deviation >10%)

5.3 Null Results in Mock Data

The Euclid Flagship v1.1 mock catalog is constructed using ΛCDM cosmology and should not exhibit the predicted harmonic features. Analysis of this mock catalog serves as a control to verify analysis methodology but is not expected to show positive detections.

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6. Relationship to Original Pre-registration

6.1 Extension Not Modification

This addendum extends rather than modifies the original pre-registered prediction. The value $\lambda_{13} = 0.856$ Mpc remains unchanged. The additional predictions for λ_{12} and λ_{14} are natural consequences of the same theoretical framework documented in the original materials.

6.2 Increased Falsifiability

The extended predictions make the framework more falsifiable by providing additional independent tests. A theory that predicts three specific scales with defined ratios faces more stringent constraints than one predicting a single scale.

6.3 Timing and Integrity

This addendum is published on 23 November 2025, before access to Euclid DR1 observational data. The Euclid Flagship v1.1 mock catalog analysis (18.9 million galaxies) is in progress but results have not yet been

examined at the time of publication. This timing ensures that the extended predictions remain genuine pre-registrations.

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7. Mathematical Appendix

7.1 Golden Ratio Properties

The golden ratio φ satisfies:

$$\varphi = (1 + \sqrt{5})/2 \approx 1.618033988749...$$
$$\varphi^2 = \varphi + 1$$
$$1/\varphi = \varphi - 1$$

7.2 Harmonic Scale Table (n = 2 to 14)

n	λ_n (analytical)	λ_n (numerical)	Regime
2	$\lambda_2 \times \varphi^0$	4.30 kpc	Galaxy
3	$\lambda_2 \times \varphi^1$	6.96 kpc	Galaxy
4	$\lambda_2 \times \varphi^2$	11.26 kpc	Galaxy halo
...	...	...	...
12	$\lambda_2 \times \varphi^{10}$	0.538 Mpc	Sub-cluster
13	$\lambda_2 \times \varphi^{11}$	0.856 Mpc	Cluster
14	$\lambda_2 \times \varphi^{12}$	1.385 Mpc	Supercluster

7.3 Uncertainty Propagation

The uncertainty in predicted values arises primarily from the measurement uncertainty in λ_2:

$$\Delta\lambda_n / \lambda_n = \Delta\lambda_2 / \lambda_2$$

With $\lambda_2 = 4.30 \pm 0.15$ kpc (from SPARC analysis), the relative uncertainty is ~3.5%, yielding:

$$\lambda_{12} = 0.538 \pm 0.019 \text{ Mpc}$$
$$\lambda_{13} = 0.856 \pm 0.030 \text{ Mpc}$$
$$\lambda_{14} = 1.385 \pm 0.048 \text{ Mpc}$$

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8. Discussion

8.1 Theoretical Self-Consistency

The harmonic scale hierarchy is a direct consequence of the 6D geometric framework. Detection of any single scale would provide support for the theory, but detection of multiple scales with the predicted φ ratio would

constitute stronger evidence for the geometric origin.

8.2 Comparison with Other Approaches

Alternative theories of modified gravity or dark matter typically do not predict discrete characteristic scales in the cosmic web regime. The specific prediction of three scales with golden ratio spacing provides a distinctive signature.

8.3 Future Observational Tests

Beyond Euclid, future surveys including DESI Year-5 data and Roman Space Telescope observations will provide independent tests of the harmonic scale predictions across different redshift ranges and tracer populations.

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9. Summary

This addendum documents the complete harmonic scale hierarchy (λ_{12}, λ_{13}, λ_{14}) predicted by the 6D geometric framework, extending the original pre-registered prediction of $\lambda_{13} = 0.856$ Mpc. The three scales follow the golden ratio progression $\lambda_n = \lambda_2 \times \varphi^{n-2}$, derived from dimensional reduction of the 6D geometry. Detection of any single scale would support the framework; detection of multiple scales with ratios consistent with φ would provide stronger evidence for the geometric origin. This addendum is published before examining Euclid DR1 observational data to maintain pre-registration integrity.

References

1. Paper I: Mathematical Foundations of 3D+3D Discrete Spacetime Theory
2. Paper II: Technical Derivations and Screening Mechanism
3. Paper V: Cosmic Web Predictions and DESI Analysis
4. Original pre-registration: $\lambda_{13} = 0.856$ Mpc prediction (published [date])

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